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AN ESTIMATE OF THE RADIUS OF THE VISCOUS CORE OF A VORTEX†

E. L. AMROMIN

St Petersburg

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A formula is derived which relates the radius of a vortex in a viscous fluid to its kinetic energy of turbulence.

We shall call the part of a vortex, within which the velocity of rotation of the fluid decreases on approaching its centre, the viscous core. The value of the nominal radius R of a vortex is used both in computational methods [1] which operate with discrete vortices and in estimating the dimensions of vortices and cavitation caverns in them in the case of the steady flows which are observed in practice [2, 3]. However, in such flows, R is a quantity that is practically independent of time while the relation between R and the circulation of a vortex Γ and the kinematic viscosity ν derived from the Navier–Stokes equations is time-dependent and does not yield a finite limit for R.

To some extent, a similar situation is also observed in the case of charged particle trajectories which are not closed in a constant electric field. However, closed trajectories do exist [5] in a pulsed field. In a real fluid the velocity field always has a pulsating part, and the availability of Reynolds equations facilitates the use of the hint in [5] that the averaged pulsation characteristics can be considered directly. In simple cases, it is even possible to derive analytic expressions for R.

Let us consider the equation for the azimuthal component of the momentum in the coordinates $\{r, \theta\}$ of a plane which is orthogonal to the axis of the vortex. The Reynolds average of this equation has the form

$$u\frac{\partial u}{\partial r}+\frac{u\upsilon}{r}+\frac{u\partial u}{r\partial\theta}+\left\langle u'\frac{\partial u'}{\partial r}\right\rangle+\left\langle\frac{u'\upsilon'}{r}\right\rangle+\frac{1}{r}\left\langle u'\frac{\partial u'}{\partial\theta}\right\rangle=-\frac{\partial p}{r\rho\partial\theta}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{r\partial r}-\frac{u}{r^2}-\frac{2\partial\upsilon}{r\partial\theta}\right) \tag{1}$$

If $\partial u/\partial\theta$, $\partial\upsilon/\partial\theta$, $\partial p/\partial\theta$ are negligibly small and the turbulence characteristics vary only slightly close to the core, then (1) can be simplified to

$$\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{r\partial r}-\frac{u}{r^2}=\frac{\langle u'\upsilon'\rangle}{\nu r}$$

Assuming $\langle u'\upsilon'\rangle \simeq$ const, the solution of this equation has the form

$$u(r)=C_1 r+\frac{C_2}{r}+\frac{\langle u'\upsilon'\rangle}{2\nu}r\ln r$$

The conditions $u(0) = 0$, $u(R) = \Gamma/(2\pi R)$, $\partial u/\partial r(R) = 0$ enable us to find the constants C_1, C_2, R, and the last of these conditions is customary for the outer boundary of a viscous layer. As a result, we obtain

$$R^2=-\Gamma\nu/\pi\langle u'\upsilon'\rangle \tag{2}$$

The above-mentioned assumptions concerning the nature of the turbulence enable us to express $\langle u'\upsilon'\rangle$ in (2) directly in terms of the kinetic energy of turbulence k. Subject to the simplifications used, $k \simeq 3\langle u'\upsilon'\rangle/2$. However, in the formula

$$R = A\sqrt{\Gamma\nu/k} \tag{3}$$

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the constant A will obviously not differ greatly from 0.69 in the case of real flows in spite of the simplifying assumptions made when deriving (3). We have not been able to verify this formula using well-known measurements since we have not found any experimental data on the values for k for vortices with measured $\{R, \Gamma\}$, and the oscillograms in [6] only enable one to assert that (3) predicts the true order of R. Equation (3) satisfies the limiting cases. In the case of a viscous fluid ($\nu \to 0$) $R \to 0$. When $k \to 0$ for finite ν, no time-independent bounded R exists as in a laminar flow. In the case of a turbulent boundary layer, the vortices must be larger in this external part where k is smaller, which corresponds to observations [7].

In conclusion, it should be noted that velocity pulsations and vortices are both attributes of turbulence, and a coupling between them is completely natural.

REFERENCES

1. SARPKAYA T., Computational methods with vortices. *Trans. ASME. J. Fluid Eng.* **111,** 1, 5–52, 1983.
2. McCORMICK B. W., On cavitation produced by a vortex trailing from a lifting surface. *Trans. ASME. Ser. D. J. Basic Eng.* **84,** 3, 369–376, 1962.
3. BILLET M. S. and HOLL J. W., Scale effects on various types of limited cavitation. *Trans. ASME. J. Fluid Eng.* **103,** 405–414, 1981.
4. BATCHELOR G., *Introduction to Fluid Dynamics*. Mir, Moscow, 1973.
5. ZASLAVSKII G. M. and SAGDEYEV R. Z., *Introduction to Non-linear Physics.* Nauka, Moscow, 1988.
6. GREEN S. I. and ACOSTA A. J., Unsteady flow in trailing vortices. *J. Fluid Mech.* **227,** 107–134, 1991.
7. CANTWELL, B. J., Organized motion in turbulent flow. *Ann. Rev. Fluid Mech.* **13,** 457–515, 1981.

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